LIST OF POSSIBLE TOPICS FOR FINAL PRESENTATIONS
RIEMANN SURFACES, SPRING 2026

Instructions: (Please read carefully)

- Students seeking a grade must deliver a final presentation on **June 9 or 16 (Weeks 15–16)**.
- Each presentation is **45 minutes** long, including time for questions from the instructor and peers. The presentation score constitutes 60% of the final grade (the remaining 40% comes from the four homework assignments) and reflects both the quality of the presentation and the depth of understanding demonstrated.
- Each student must choose a distinct topic from the list below, or propose an alternative topic with the instructor's approval. Topic selection must be discussed with the instructor **before April 30** to allow sufficient preparation time.
- **Note:** The list of topics may be updated to include additional options.
- **Note:** In your presentation, you should:
  - Provide precise definitions for any notions not introduced in class.
  - State explicitly and precisely the statements you intend to prove.
  - Give detailed proofs of those statements.
  - Ensure you fully understand all the material you present, as questions will be asked.
- **Note:** Topics with a (*) might be more difficult.

0.1. **Topic: Riemann's existence theorem.** State and prove Riemann's existence theorem, concerning the realization of monodromy by a branched cover (we discussed briefly in class).

Reference: Donaldson, Riemann Surfaces, Chapter 4.2.2; Miranda, Algebraic Curves and Riemann Surfaces, Chapter III.4.

0.2. **Topic: Maximal analytic continuation.** Let X be a Riemann surface, $a \in X$, and $\varphi \in \mathcal{O}_a$ a germ of holomorphic function at a. Define *analytic continuation* of φ, and prove that a *maximal analytic continuation* exists.

Reference: Forster, Lectures on Riemann Surfaces, Chapter 7.

0.3. **Topic: Galois correspondence of covering maps.** Let X be a compact Riemann surface. Prove that there is an equivalence of categories between the category of field extensions of $\mathcal{M}(X)$ and the category of branched coverings of compact Riemann surfaces over X.

Reference: Forster, Lectures on Riemann Surfaces, Chapter 8; Donaldson, Riemann Surfaces, Chapter 11.1.

0.4. **Topic: Non-compact version of Hodge theorem.** Let X be a connected, simply connected, non-compact Riemann surface. Suppose ρ is a real 2-form of compact support on X with $\int_X \rho = 0$. Prove that there exists a (real-valued) function ϕ on X with $\Delta\phi = \rho$ and such that ϕ tends to 0 at infinity in X.

Reference: Donaldson, Riemann Surfaces, Chapter 10.2.

0.5. **Topic: Vector bundles on $\mathbb{P}^1$.** Prove that every holomorphic vector bundle on $\mathbb{P}^1$ is isomorphic to a direct sum of line bundles (Grothendieck's theorem.)

Reference: Many online sources can be found.

0.6. **Topic(*): Klein quartic curve.** Prove that the Klein quartic curve

$$\{x^3y + y^3z + z^3x = 0\} \subseteq \mathbb{CP}^2$$

has an automorphism group of order 168, attaining the Hurwitz bound. Prove further that it is isomorphic to the modular curve associated with the principal congruence subgroup $\Gamma(7) \subseteq \mathrm{PSL}(2, \mathbb{Z})$.

Reference: Many online sources can be found, e.g. Elkies, The Klein Quartic in Number Theory.

0.7. **Topic(*): Spectral sequences.** Introduce spectral sequences and provide an example of their use in computing sheaf cohomology, e.g. $H^k(M, \underline{\mathbb{R}}) \cong H^k_{\mathrm{dR}}(M)$, where $\underline{\mathbb{R}}$ denotes the $\mathbb{R}$-valued constant sheaf on M.

Reference: Many online sources can be found, e.g. this note.

0.8. **Topic(*): Picard–Fuchs equations for periods of elliptic curves.**

Reference: Schnell, On computing Picard–Fuchs equations.

0.9. **Topic(*): Belyi's theorem.** A compact Riemann surface X is an algebraic curve defined over $\overline{\mathbb{Q}}$ if and only if there exists a non-constant holomorphic map $f\colon X \to \mathbb{C}$ whose critical values lie in $\{0, 1, \infty\}$.

Reference: Many online sources can be found, e.g. this note.