

In this course, we will primarily focus on the bounded derived category of coherent sheaves $D^b\text{Coh}(X)$ on a smooth projective variety X . Occasionally, we will also explore other triangulated categories, such as the derived Fukaya category of a symplectic manifold and the derived category of representations of a quiver.

The aim of this course is to introduce concepts related to extracting geometric information from the derived category of X , including birational geometry and mirror symmetry of Calabi–Yau manifolds, as well as mirror symmetry between Fano manifolds and Landau–Ginzburg models. To achieve this, we will study the group of autoequivalences $\text{Aut}(D^b(X))$ and the space of Bridgeland stability conditions $\text{Stab}(D^b(X))$.

For different types of varieties (Fano, Calabi–Yau, or general type), different questions arise, and the tools used to address these questions vary. These questions are at the frontier of current research, many of which are suitable for graduate students to pursue as thesis projects. We can select the topics to be discussed based on audience preferences.